A NOTE TO THE HEAT TRANSFER FROM A MOVING FIBRE

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Received May 3rd, 1971

The method to find the average Nusselt number for isothermal continuous cylinder penetrating an isothermal fluid environment has been described to complete the paper of Bourne and Elliston.

Bourne and Elliston[1] have recently presented a solution of the boundary layer on a continuous isothermal cylinder penetrating an isothermal fluid of infinite extent. The problem has been solved by Kármán–Pohlhausen integral technique and the results of numerical computation for Prandtl number $\sigma \leqq 1$ have been given in the form of the relation of the local Nusselt number Nu and the dimmensionless distance X from the origin of the cylinder. Nevertheless, from the practical point of view the knowledge of the average Nusselt number $\overline{\mathrm{Nu}}$ is more interesting. In the present paper, the original work of Brown and Elliston[1] has been completed in this way. The method of calculation of $\overline{\mathrm{Nu}}$ originates in a previous work of the author[2]. The heat flow Q from the cylinder of length x, defined by

$$Q = 2\pi a x \bar{h}(T_w - T_\infty) \qquad (1)$$

raises the temperature of the fluid in the boundary layer from T_∞ to T:

$$Q = 2\pi \varrho c_p \int_a^\infty u(T - T_\infty)\, r\, \mathrm{d}r\,. \qquad (2)$$

In the same way as displacement or momentum thickness and area[3], the heat thickness μ and heat area M can be defined:

$$M = \pi[(a + \mu)^2 - a^2]\,, \qquad (3)$$

$$Q = M \varrho U c_p (T_w - T_\infty)\,. \qquad (4)$$

From Eqs (2) and (4) follows:

$$M = [2\pi/U(T_w - T_\infty)] \int_a^\infty u(T - T_\infty)\, r\, \mathrm{d}r\,. \qquad (5)$$

The integral on the right side of Eq. (*5*) is identical with the integral in Eq. (*18*) of the paper cited[1]; integrating yields for the dimmensionless heat area (the detailed course of integration can be found in reference[2])

$$M/\pi a^2 = [(\beta - \alpha + 1) \exp(2\alpha) - (\alpha + 2\alpha\beta + \beta + 1)] (2\alpha\beta)^{-1} . \qquad (6)$$

The average Nusselt number $\overline{\mathrm{Nu}}$:

$$\overline{\mathrm{Nu}} \equiv \bar{h}a/k = Q/2\pi x(T_w - T_\infty) k \qquad (7)$$

can be expressed with the aid of the heat area as

$$\overline{\mathrm{Nu}} = (M/\pi a^2) a^2 \varrho U c_p/2kx . \qquad (8)$$

Table I

Parameters α and β of the Boundary Layer, Heat Area and Average Nusselt Numbers for $\sigma = 0{\cdot}72$

log X	α	β	$M/\pi a^2$	$\overline{\mathrm{Nu}}$	$2\pi\overline{\mathrm{Nu}}$
−4	0·02430	0·03056	0·01810	65·16	409·3
−3·5	0·04294	0·05394	0·03230	36·77	231·0
−3	0·07552	0·09471	0·05781	20·81	130·8
−2·5	0·1317	0·1647	0·1040	11·84	74·37
−2	0·2267	0·2818	0·1886	6·789	42·66
−1·5	0·3814	0·4706	0·3470	3·950	24·82
−1	0·6202	0·7559	0·6531	2·351	14·77
−0·5	0·9621	1·153	1·273	1·449	9·105
0	1·409	1·655	2·602	0·9366	5·885
0·5	1·941	2·233	5·635	0·6415	4·031
1	2·528	2·863	13·03	0·4692	2·948
1·5	3·143	3·490	31·62	0·3600	2·262
2	3·769	4·129	80·81	0·2909	1·828
2·5	4·397	4·766	214·3	0·2439	1·533
3	5·024	5·398	584·1	0·2103	1·321
3·5	5·647	6·025	1 626	0·1851	1·163
4	6·267	6·647	4 598	0·1655	1·040
4·5	6·884	7·265	$1{\cdot}317 . 10^4$	0·1499	0·9418
5	7·498	7·880	$3{\cdot}807 . 10^4$	0·1370	0·8611
5·5	8·109	8·492	$1{\cdot}110 . 10^5$	0·1264	0·7941
6	8·717	9·102	$3{\cdot}255 . 10^5$	0·1172	0·7364
6·5	9·324	9·709	$9{\cdot}605 . 10^5$	0·1093	0·6870
7	9·929	10·31	$2{\cdot}847 . 10^6$	0·1025	0·6441

Introducing the dimmensionless distance $X = \nu x/Ua^2$ from the origin of the cylinder, it follows:

$$\overline{\mathrm{Nu}} = (M/\pi a^2)\,\sigma/2X\,. \qquad (9)$$

The Nusselt number in the paper[1] has been defined with the aid of the heat flow per unit length of the cylinder. To get the average Nusselt number $\overline{\mathrm{Nu}}$ expressed in the same way, $\overline{\mathrm{Nu}}$ defined by Eq. (*9*) is to be multiplied by the value 2π.

The values of α, β, $(M/\pi a^2)$, $\overline{\mathrm{Nu}}$ and $2\pi\overline{\mathrm{Nu}}$ for the practically important case of Prandtl number $\sigma = 0{\cdot}72$ are displayed in Table I for equidistant values of $\log X$ to allow linear interpolation. The values of α for various X have been calculated with the aid of the HP 9100A calculator, using the integrated Sakiadis equation[2,4], identical with Eq. (*12*) in the original paper[1]:

$$X = \{[\exp(2\alpha) - 1]/2\alpha\} - 1 - 0{\cdot}5 \sum_{m=1}^{\infty} \frac{(2\alpha)^m}{m\,.\,m!}\,. \qquad (10)$$

For the calculation of β from the equation

$$\frac{\mathrm{d}\beta}{\mathrm{d}\alpha} = \frac{\beta}{\alpha}\left[\frac{2}{\sigma} + \frac{(2\alpha^2 - 2\alpha + \beta - 2\alpha\beta + 1)\exp(2\alpha) - \beta - 1}{(\alpha - 1)\exp(2\alpha) + \alpha + 1}\right], \qquad (11)$$

given in the previous work[2] and identical with the Eq. (*23*) in the paper of Bourne and Elliston[1], the third-order Runge–Kutta procedure was used with the help of a HP 9100B calculator.

LIST OF SYMBOLS

a radius of cylinder
A surface area of cylinder
c_p specific heat at constant pressure of the fluid
$\bar{h} = Q/A(T_w - T_\infty)$ average heat transfer coefficient
k thermal conductivity of the fluid
M heat area, Eqs. (*3*), (*4*)
$\mathrm{Nu} = Q/kx(T_w - T_\infty)$ local Nusselt number
$\overline{\mathrm{Nu}} = \bar{h}a/k$ average Nusselt number
Q heat flow
r radial coordinate
T temperature
T_w surface temperature of the cylinder
T_∞ ambient temperature of the fluid
U speed of the cylinder
u axial fluid velocity
x axial coordinate (distance from the origin of the cylinder)
$X = \nu x/Ua^2$ dimensionless distance from the origin of the cylinder
α parameter in boundary layer velocity profile, Eq. (*7*) of ref.[1]
β parameter in boundary layer temperature profile, Eq. (*19*), ref.[1]
μ heat thickness, Eq. (*3*)

ν kinematic viscosity of the fluid
ϱ density of the fluid
$\sigma = \nu c_p \varrho / k$ Prandtl number

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Translated by the author.